QUIZ TODAY!

Special Triangles
and the 6 trig functions

20 points

No calculator!

Previous notes from 6.3 (part 1)

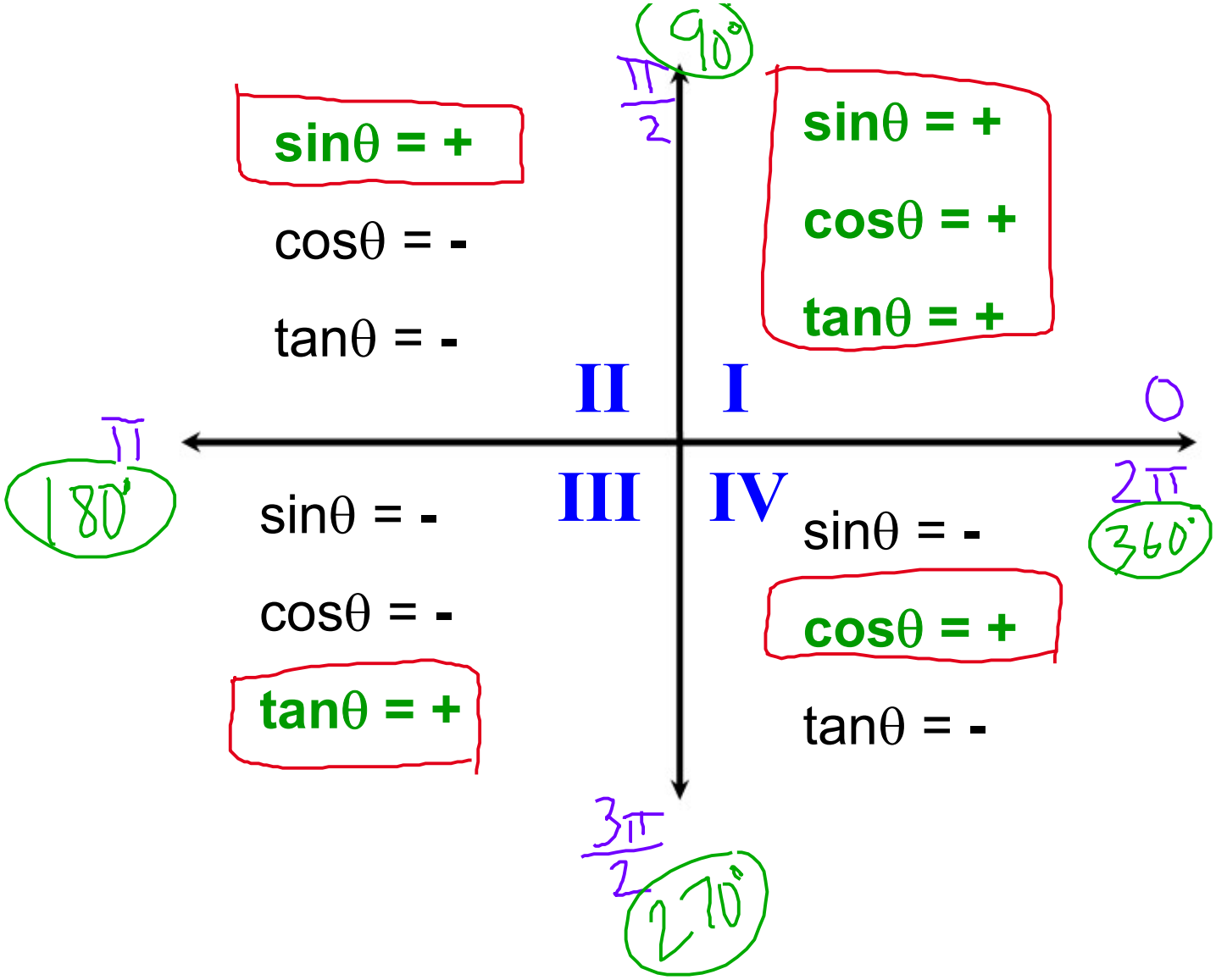
Coterminal Angles

$\theta \pm 360°(n)$ or $\theta \pm 2\pi(n)$

$0° \le \theta < 360°$ $0 \le \theta < 2\pi$

QUADRANT	REFERENCE ANGLE (DEGREES)	REFERENCE ANGLE (RADIANS)
1	θ	θ
2	$180° - \theta$	$\pi - \theta$
3	$\theta - 180°$	$\theta - \pi$
4	$360° - \theta$	$2\pi - \theta$

Previous notes from 6.3 (part 1)

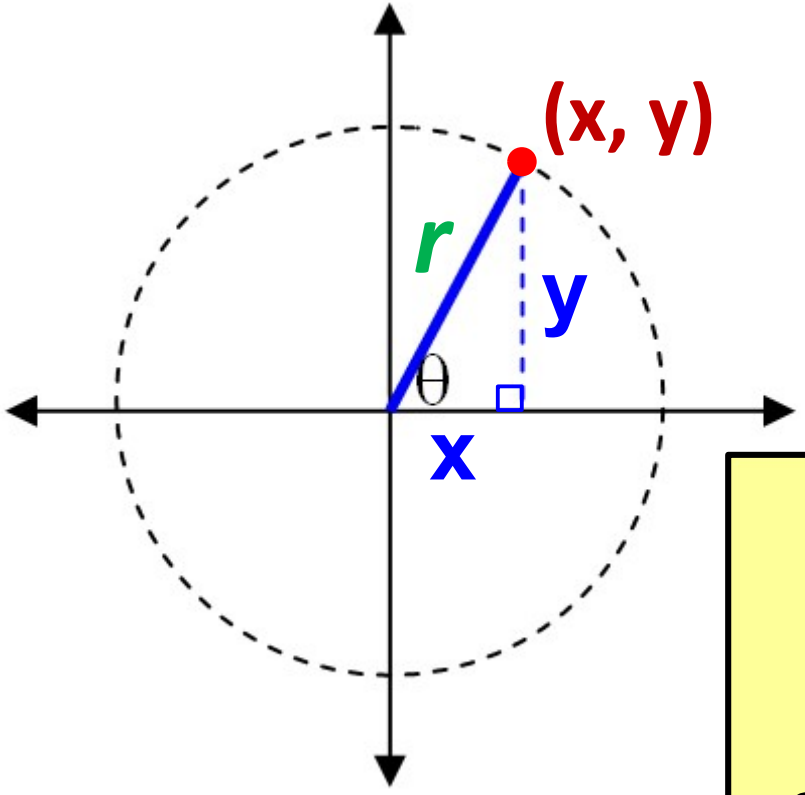


Notes: 6.3 part 2 → NEW!

Any point on a circle with radius ***r*** *has coordinates (x,y) where* $r^2 = x^2 + y^2$

or

$r = \sqrt{x^2 + y^2}$



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

IMPORTANT: θ is the angle closest to the origin and always formed with the X-AXIS

Notes: 6.3 part 2

$$r^2 = x^2 + y^2$$

or

$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

IMPORTANT: θ **is the angle closest to the origin and <u>always</u> formed with the X-AXIS**

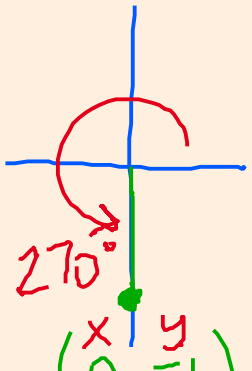
Sine
Cosine
Tangent

Cosecant
Secant
Cotangent

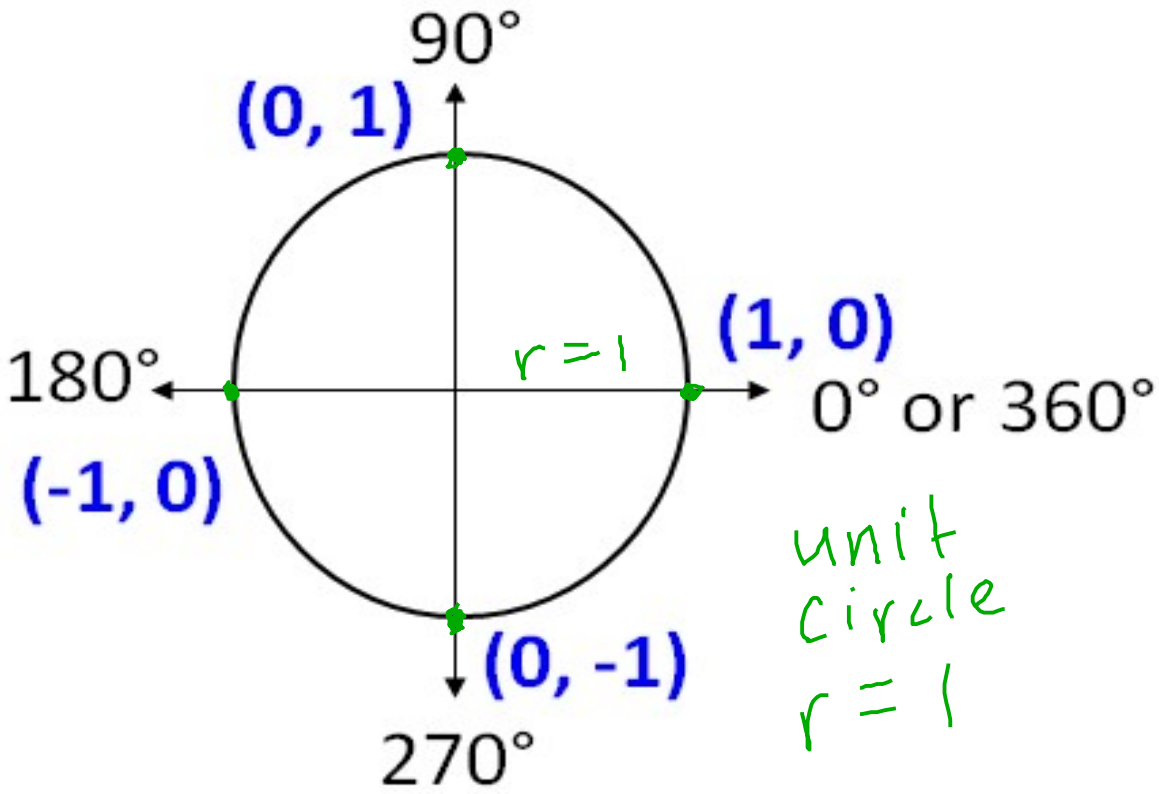
Notes: **Some special angles are on boundary lines (triangle NOT possible.)**

Example:

$$\sin 270° = \frac{y}{r} = \frac{-1}{1} = -1$$



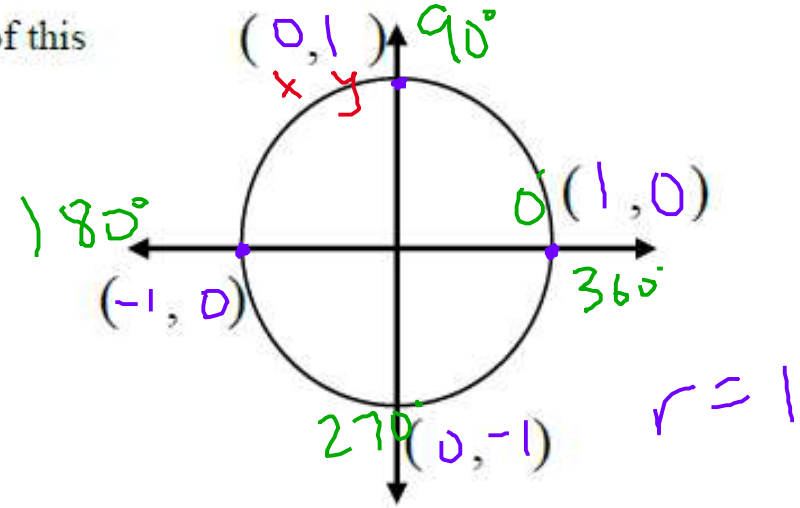
always positive



unit circle
r = 1

Warm-up: Solve using special angles that are located on the boundary lines:

WARM-UP: Label the degrees and coordinates on the axes of this unit circle, then find the solutions for a-k.



	definition	ratio	solution
a) tan 180° =	$\frac{y}{x}$ =	$\frac{0}{-1}$ =	0
b) tan 90° =	$\frac{y}{x}$ =	$\frac{1}{0}$ =	undef
c) sin 0° =			
d) cos 0° =			
e) sec 180° =			
f) csc 0° =			
g) cos 90° =			
h) sin 270° =			
i) sec 270° =			
j) cot 90° =			
k) csc 90° =			

CHECK ANSWERS a-k

0 0 0 0
-1 -1 1 1
~~undefined~~
undefined
undefined

Check answers to warm-up

a) 0

b) undefined

c) 0

d) 1

e) -1

f) undef

g) 0

h) -1

i) undef

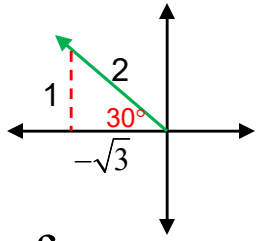
j) 0

k) 1

- Write the given problem for #13-24.
- State its trig definition in terms of x, y, r.
- State the quadrant.
- Find reference angle.
- Sketch diagram, label sides using special triangles, then solve.
- See example on check answer/hint sheet.

13. cos150°

$\cos 150° = \frac{x}{r}$



Quadrant II

180°-150°=30° reference angle

So $\cos 150° = \boxed{-\frac{\sqrt{3}}{2}}$

	$0° \le \theta < 360°$	$0 \le \theta < 2\pi$
QUADRANT	REFERENCE ANGLE (DEGREES)	REFERENCE ANGLE (RADIANS)
1	θ	θ
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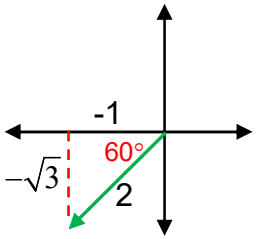
CHECK #13-24 (in random order)

TRIG DEFINITION	QUAD	REF ANGLE	SOLUTION
$\frac{x}{y}$ $\frac{x}{y}$	I	30 30 30	$-\frac{2\sqrt{3}}{3}$ $-\frac{\sqrt{3}}{3}$
$\frac{x}{r}$ $\frac{x}{r}$ $\frac{x}{r}$	II II	30 30 30	$-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2}$
$\frac{y}{r}$ $\frac{y}{r}$	III III III III	60 60 60	-2 $-\frac{1}{2}$ $\frac{1}{2}$ 1
$\frac{r}{x}$ $\frac{r}{y}$ $\frac{r}{y}$	IV IV IV IV	60 60	$\frac{\sqrt{3}}{3}$ $\frac{\sqrt{3}}{3}$ $\sqrt{3}$
$\frac{y}{x}$ $\frac{y}{x}$	boundary line	none	

14. sin240°

$\sin 240° = \frac{y}{r}$

Quadrant III



240°-180°=60° reference angle

So $\sin 240° = -\frac{\sqrt{3}}{2}$

CHECK #13-24 (in random order)

TRIG DEFINITION	QUAD	REF ANGLE	SOLUTION
$\frac{x}{y}$ $\frac{x}{y}$	I	30 30 30	$-\frac{2\sqrt{3}}{3}$ $-\frac{\sqrt{3}}{3}$
$\frac{x}{r}$ $\frac{x}{r}$ $\frac{x}{r}$	II II	30 30 30	$-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2}$
$\frac{y}{r}$ $\frac{y}{r}$	III III III III	60 60 60	-2 $-\frac{1}{2}$ $\frac{1}{2}$ 1
$\frac{r}{x}$ $\frac{r}{y}$ $\frac{r}{y}$	IV IV IV IV	60 60	$\frac{\sqrt{3}}{3}$ $\frac{\sqrt{3}}{3}$ $\sqrt{3}$
$\frac{y}{x}$ $\frac{y}{x}$	boundary line	none	

Note: if angles are smaller than 0° or larger than 360°, then find the coterminal angle first by adding or subtracting 360n.

15. tan(330°)

16. sin(-30°)

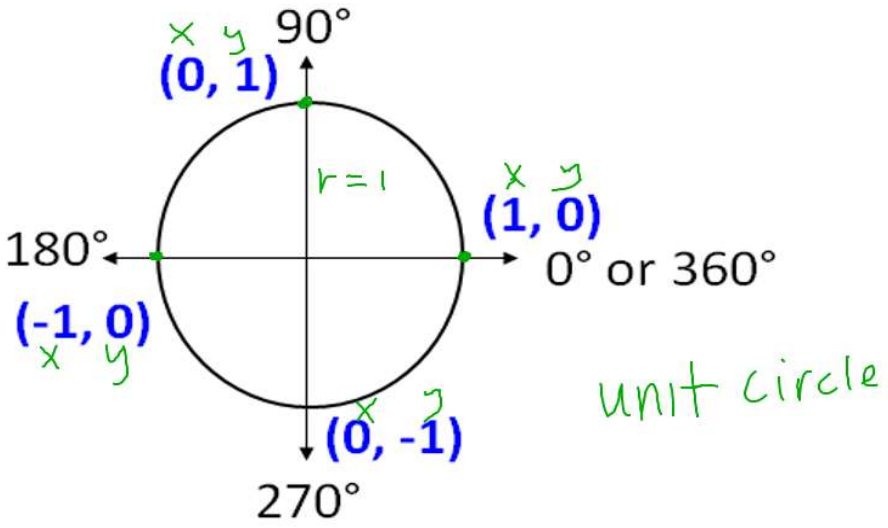
17. cot(-120°)

USEFUL TOOLS

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



$$r^2 = x^2 + y^2$$

Coterminal Angles

$\theta \pm 360°(n)$ or $\theta \pm 2\pi(n)$

$0° \le \theta < 360°$ $\quad 0 \le \theta < 2\pi$

QUADRANT	REFERENCE ANGLE (DEGREES)	REFERENCE ANGLE (RADIANS)
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USEFUL TOOLS: